The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 1: Sequences, Unit 1 Introduction

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Identify and work with composed functions
- Decompose complex composed functions into their simplest components
- Identify and use proper notation for finite sums and products.

Understanding(s):
Students will understand …

- Functions
    - Composition of functions as substitution of one function into another function.
    - Decomposition of composed functions.
- Sums and Products
    - How to use proper notation for finite sums.
    - How to use proper notation for finite products.

Essential Question(s):

- What are functions?
- How do we substitute constants into functions?
- What are the general ways to represent sums and products?

Knowledge:
Students will know …

- Functions and their properties
- Composition of functions
- Some properties of finite sums and products

Skills:
Student will be able to:

- Identify and work with composed functions
- Decompose complex composed functions into their simplest components
- Identify and use proper notation for finite sums and products.

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly

Other Evidence:

The following will also be observed, recorded, and

recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30% of student grades. • Comprehension (true/false, definitions, identifying topics and themes, etc.) • Solving pure mathematical problems as well as word problems. • Discussions and presentations • Group project that involves research and report writing • Homework assignments	considered for the final grade of students in each lesson activity • Motivation • Engagement • Collaboration • Communication pattern among peers and with the teacher • Reactions • Respect to others and different opinions

Stage 3 – Learning Plan

Learning Activities:

In this course, students are involved in a variety of class activities to understand mathematics at a deeper level, to transfer their knowledge to other contexts, and to improve their skills of working with mathematics in the form of discussion, presentation, and interaction. In so doing, students demonstrate their ability to use English mathematical language and notation appropriate to their grade level. The following is a summary of lesson activities for the course.

1. Individual/pair/small group activity

Students practice and improve solving pure mathematical problems for the general topic, looking for connections with previous topics, using notation and terminology, identifying a sequence to solve a problem, inferring mathematics from written English, and solving real-world problems.

2. Discussion and presentation:

Students in pair or in small groups will discuss a topic or an issue given. After a certain time, they will share their ideas with the class. This activity will boost student imagination and creativity, help them understand that mathematics is more than calculating, and improve cooperation and collaboration with peers.

3. Critical Thinking Activities

Students are involved in more challenging discussions and activities at grade level that are related to higher-order thinking skills according to the revised Bloom's Taxonomy as below:

Applying

Students can apply their knowledge on mathematical concepts to other contexts in their lives after reading. Student application of their knowledge will be demonstrated during the class activities, such as discussion, presentation, peer-review, and problem-solving.

Analyzing

Students can compare and contrast different methods for solving problems. Students also analyze different types of problems without a clearcut solution laid out for them. Finally, students will also analyze their peers board work and presentations. Students will gain an appreciation for peer-review, which is a fundamental element of both mathematics and science.

Evaluating

Students can evaluate possible solutions to a problem and settle on the one that will best solve the problem at hand. Students will also evaluate the work of their peers and suggest alternative methods for solving problems. In doing so, students will gain a deeper understanding and appreciation for mathematics and mathematical thinking.

Creating

Students can demonstrate their creativity and imagination by working challenging problems based on their lesson. Some activities will involve solving real-world problems from a variety of disciplines. These problems will be different from the standard word problem as they will draw on the student's ability to link various aspects of mathematics together in order to solve the problem.

The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 1: Sequences, Unit 2 Combinatorics

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Explain why factorials grow very large very quickly. Example: $100! > 9.3326 \times 10^{157}$
- Explain how to simplify Permutations and Combinations to make calculating them easier.
- Explain why $P(n,n) = n!$ And explain why this is all the ways to rearrange a set of n elements.
- Discuss why functions of the form $f(x) = \frac{a^x}{x!}$, for $a \in N$ and $x \in R$ get smaller as x gets bigger.
- Discuss the problem of finding all the ways to rearrange a set of n elements. The discussion should start with the set $S_2 = \{1,2\}$. The students can see there are 2 ways to rearrange this set. Next the discussion should move to $S_3 = \{1,2,3\}$. The students should be tasked
- State Newton's binomial expansion formula.
- Use the binomial expansion formula to expand binomials of the form $(a+b)^n$
- Understand the relationship between Pascal's triangle and Newton's binomial expansion formula.
- Construct Pascal's triangle for a given value of n.

Understanding(s):
Students will understand …

- Factorials:

$$n! = 1 \cdot 2 \cdots n = \prod_{j=1}^{n} j$$

$$0! = 1! = 1$$

- Permutations:

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

- Combinations:

$$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

- Newton's binomial expansion formula
- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$
- The concept of Pascal's Triangle and that it gives the terms of Newton's binomial expansion.

Essential Question(s):

- How many ways can we rearrange the elements of a set of order n?
- How many ways can we rearrange groups of k elements from a set of size n if the order of the elements matter?
- How many ways can we arrange groups of $k$elements from a set of size n if the order of the elements doesn't matter?
- How do we easily expand binomials of the form $(a+b)^n$?

The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019

Knowledge: *Students will know …*	**Skills:** *Student will be able to:*
• Factorial. • Permutation. • Combination. • Newton's binomial theorem. • Pascal's triangle.	• Explain why factorials grow very large very quickly. Example: $100! > 9.3326 \times 10^{157}$ • Explain how to simplify Permutations and Combinations to make calculating them easier. • Explain why $P(n,n) = n!$ And explain why this is all the ways to rearrange a set of n elements. • Discuss why functions of the form $f(x) = \frac{a^x}{x!}$, for $a \in N$ and $x \in R$ get smaller as x gets bigger. • Discuss the problem of finding all the ways to rearrange a set of n elements. The discussion should start with the set $S_2 = \{1,2\}$. The students can see there are 2 ways to rearrange this set. Next the discussion should move to $S_3 = \{1,2,3\}$. The students should be tasked with discovering that there are 6 ways to rearrange the elements of S_3. S_3 is particularly interesting because it forms the symmetric group of 3 elements. The students will use this later when they learn about function composition. This discussion should lead directly into the concept of factorial. • State Newton's binomial expansion formula. • Use the binomial expansion formula to expand binomials of the form $(a+b)^n$ • Understand the relationship between Pascal's triangle and Newton's binomial expansion formula. • Construct Pascal's triangle for a given value of n.

Stage 2 - Assessment Evidence

Performance Task(s):	**Other Evidence:**
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The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 1: Sequences, Unit 3 Introduction to Sequences

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Define finite and infinite sequences.
- Determine a sequence given a general term.
- Find the general term given a sample sequence of numbers.
- Find a sequence from a description.
- Discuss recursive methods.
- Understand and calculate Fibonacci Numbers.
- Represent a finite sequence geometrically.
- Define and identify increasing, decreasing, and bounded sequences.

Understanding(s):
Students will understand …

- Let $\varphi: N \to R$ be the map $n \mapsto \varphi(n)\ \forall\, n \in N$, then we call φ an ***infinite sequence***
- Let $M = \{1, 2, 3, \cdots, m\} \subset N$ $\phi: M \to R$ be the map $m \mapsto \phi(m)\ \forall\, m \in M$, then we call ϕ a ***finite sequence***
- We often write φ, ϕ as $S(N)$
- Sequence given by
  - General term
  - Descriptive method
  - Recursive method (Fibonacci Numbers)
- Geometric representation of a sequence
- Increasing, decreasing, and bounded sequences

Essential Question(s):

- What real-world applications are there for sequences?
- Where do sequences occur in the natural world?

Knowledge:
Students will know …

- Recursive methods.
- Fibonacci Numbers.

Skills:
Student will be able to:

- Define finite and infinite sequences.
- Determine a sequence given a general term.
- Find the general term given a sample sequence of numbers.
- Find a sequence from a description.
-

Stage 2 - Assessment Evidence

The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019

Performance Task(s):	**Other Evidence:**
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Stage 3 – Learning Plan

Learning Activities:
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Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 2: Limits, Unit 1 Limits of a Sequence

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Define the limit of a sequence
- Calculate finite limits of sequences
- Calculate infinite limits of sequences
-

Understanding(s):
Students will understand …

- Definition
- Finite limits
  - Special limits
  - Theorem on finite limits
- The sum of an infinite geometric series
  - Infinite Limits
  - Special limits
- Theorem on infinite limits

Essential Question(s):

- What happens to sequences as the value of n get bigger (or smaller)?
- What happens to a sequence if we let n grow without bounds?

Knowledge:
Students will know how to …

- Calculate finite limits of sequences
- Calculate infinite limits of sequences

Skills:
Student will be able to:

- Define the limit of a sequence

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30% of student grades.

- Comprehension (true/false, definitions, identifying topics and themes, etc.)
- Solving pure mathematical problems as well as

Other Evidence:

The following will also be observed, recorded, and considered for the final grade of students in each lesson activity

- Motivation
- Engagement
- Collaboration
- Communication pattern among peers and with the teacher
- Reactions

word problems. • Discussions and presentations • Group project that involves research and report writing • Homework assignments	• Respect to others and different opinions

Stage 3 – Learning Plan

Learning Activities:

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Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 2: Limits, Unit 2 Limits of a Function

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Define the limit of a function.
- Take simple limits.
- State and use the theorems on limits.
- Define one-sided limits.
- Take one-sided limits.

Understanding(s):
Students will understand …

- If $\lim_{x \to x_0} f(x) = L$ and $\lim_{x \to x_0} g(x) = M$ then
  - $\lim_{x \to x_0} [f(x) + g(x)] = L + M$
  - $\lim_{x \to x_0} [f(x) - g(x)] = L - M$
  - $\lim_{x \to x_0} [f(x) \cdot g(x)] = L \cdot M$
  - $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{L}{M} (M \neq 0)$
- If $f(x) \geq 0$ and $\lim_{x \to x_0} f(x) = L$, then
  - $L \geq 0$
  - $\lim_{x \to x_0} \sqrt{f(x)} = \sqrt{L}$
- One-sided limits
- Infinite limits
- If $k \in N$
  - $\lim_{x \to +\infty} x^k = +\infty$
- If $k \in N$ is odd
  - $\lim_{x \to -\infty} x^k = -\infty$
- If $k \in N$ is even
  - $\lim_{x \to -\infty} x^k = +\infty$
- Rules for infinite limits

Essential Question(s):

- What happens to functions as the value of x gets close to a given number?
- What happens to functions if we let n grow without bounds?

The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019

Knowledge: *Students will know how to …* • Take simple limits. • Take one-sided limits.	**Skills:** *Student will be able to:* • Define the limit of a function. • State and use the theorems on limits. • Define one-sided limits.

Stage 2 - Assessment Evidence

Performance Task(s):	**Other Evidence:**
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Stage 3 – Learning Plan

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Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 3, Unit 1 Introduction to Derivatives

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Discuss the historical problems that led to the development of the derivative
- Understand the relationship between the secant line and the tangent line
- Derive the definition of the derivative from secant and tangent lines
- Understand the derivative as instantaneous velocity
- Recognize Leibniz's notation, Lagrange's notation, and Newton's notation
- Calculate basic derivatives using the formal definition of the derivative

Understanding(s):
Students will understand …

- The motivation that led to the discovery of the derivative.
- The advantages and disadvantaged of the various notation systems used for the derivative.
- How the derivative relates to secant and tangent lines.
- How the derivative relates to velocity problems.
- How to calculate some derivatives using the formal definition.

Essential Question(s):

- Why did Newton and Leibniz discover the derivative?
- Was the derivative discovered or invented?
- How does the derivative relate to problems in geometry?
- How does the derivative relate to problems in physics?

Knowledge:
Students will know …

- How to use secant lines and tangent lines to derive the formal definition of the derivative.
- How to calculate the derivative of simple functions using the formal definition of the derivative.
- How to solve some basic velocity problems using derivatives.

Skills:
Student will be able to:

- Describe secant lines.
- Describe tangent lines.
- Describe the formal definition of the derivative in terms of secant lines and tangent lines.
- Talk about how the derivative relates to instantaneous velocity.

Stage 2 - Assessment Evidence

Performance Task(s):

Other Evidence:

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Chapter 3, Unit 2 Rules for Calculating Derivatives

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Take the derivatives of some common functions.
- State the rules of derivatives; Constant Rule, Constant Multiple Rule, Multiple Rule, Sum/Difference Rules, Product Rule, Quotient Rule, and Chain Rule.
- Talk about composition of functions and understand why composition is non-commutative.
- Discuss the chain rule and its relationship to taking derivatives.
- Take derivatives of composite functions using the chain rule.

Understanding(s):
Students will understand …

- How to find formulas for common functions using the formal definition of the derivative.
- The Rules of Derivatives.
- How to compose two (or more) functions.
- How to recognize whether or not a function is a composite function.
- On a deeper level, that function composition is an operation similar to addition, but one that is non-commutative.
- Relationships exist between composite functions and real-world applications, particularly in physics.

Essential Question(s):

- Are there any short cuts to taking derivatives, or must we always use the formal definition?
- Are there methods available for finding derivatives for functions such as
  - $f(x) = \sqrt{x^2 + x}$
  - $f(x) = \left(\frac{8x - x^6}{x^3}\right)^{\frac{-4}{5}}$
  - $f(x) = cos^2(x^3 - 1)$
- What special properties do composite functions have?
- Is function composition similar to operations on the real numbers?

Knowledge:
Students will know …

- Formulas for calculating the derivatives of common functions.
- The Basic Rules of Derivatives
  a. $\frac{d}{dx}(c) = 0$
  b. $\frac{d}{dx}(af(x)) = a\frac{d}{dx}f(x)$

Skills:
Student will be able to:

- State the formulas for finding the derivative of common functions.
- Correctly use the formulas to find the derivatives of basic functions.
- State and use the basic rules of derivatives.
- Discuss function composition.

c. $\frac{d}{dx}f(ax) = a\frac{d}{du}f(u), u = ax$

d. $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

e. $\frac{d}{dx}f(x)g(x) = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$

f. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$

- How function composition works and why it is important, not just to math, but to other subjects as well.
- How to take derivative using the chain rule

a. $\frac{d}{dx}f(g(x)) = \left(\frac{d}{du}f(u)\right)\frac{d}{dx}g(x), u = g(x)$

- State and use the chain rule.

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30% of student grades.

- Comprehension (true/false, definitions, identifying topics and themes, etc.)
- Solving pure mathematical problems as well as word problems.
- Discussions and presentations
- Group project that involves research and report writing
- Homework assignments

Other Evidence:

The following will also be observed, recorded, and considered for the final grade of students in each lesson activity
Motivation
Engagement
Collaboration
Communication pattern among peers and with the teacher
Reactions
Respect to others and different opinions

Stage 3 – Learning Plan

Learning Activities:
In this course, students are involved in a variety of class activities to understand mathematics at a deeper level, to transfer their knowledge to other contexts, and to improve their skills of working with mathematics in the form of discussion, presentation, and interaction. In so doing, students demonstrate their ability to use English mathematical language and notation appropriate to their grade level. The following is a summary of lesson

activities for the course.

1. Individual/pair/small group activity

Students practice and improve solving pure mathematical problems for the general topic, looking for connections with previous topics, using notation and terminology, identifying a sequence to solve a problem, inferring mathematics from written English, and solving real-world problems.

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3. Critical Thinking Activities

Students are involved in more challenging discussions and activities at grade level that are related to higher-order thinking skills according to the revised Bloom's Taxonomy as below:

Applying

Students can apply their knowledge on mathematical concepts to other contexts in their lives after reading. Student application of their knowledge will be demonstrated during the class activities, such as discussion, presentation, peer-review, and problem-solving.

Analyzing

Students can compare and contrast different methods for solving problems. Students also analyze different types of problems without a clearcut solution laid out for them. Finally, students will also analyze their peers board work and presentations. Students will gain an appreciation for peer-review, which is a fundamental element of both mathematics and science.

Evaluating

Students can evaluate possible solutions to a problem and settle on the one that will best solve the problem at hand. Students will also evaluate the work of their peers and suggest alternative methods for solving problems. In doing so, students will gain a deeper understanding and appreciation for mathematics and mathematical thinking.

Creating

Students can demonstrate their creativity and imagination by working challenging problems based on their lesson. Some activities will involve solving real-world problems from a variety of disciplines. These problems will be different from the standard word problem as they will draw on the student's ability to link various aspects of mathematics together in order to solve the problem.

The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 3, Unit 3 Derivatives of the Trigonometric Functions

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Quickly state the derivatives of the basic trig functions
- Use the derivatives of the basic trig functions to solve word problems.

Understanding(s):
Students will understand …

- Derivatives of
  - $f(x) = \sin x$
  - $f(x) = \cos x$
  - $f(x) = \tan x$
  - $f(x) = \cot x$
  - $f(x) = \sec x$ (Supplement)
  - $f(x) = \csc x$ (Supplement)

Essential Question(s):

- How do we take the derivative of the six basic trig functions?
- How can the derivative of trig functions help us solve real-world problems?

Knowledge:
Students will know …

- How to set up word problems involving the derivative of the trigonometric functions.

Skills:
Student will be able to:

- Quickly state the derivatives of the basic trig functions
- Use the derivatives of the basic trig functions to solve word problems.

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30% of student grades.

- Comprehension (true/false, definitions, identifying topics and themes, etc.)
- Solving pure mathematical problems as well as word problems.

Other Evidence:

The following will also be observed, recorded, and considered for the final grade of students in each lesson activity

- Motivation
- Engagement
- Collaboration
- Communication pattern among peers and with the teacher
- Reactions
- Respect to others and different opinions

• Discussions and presentations • Group project that involves research and report writing • Homework assignments	

Stage 3 – Learning Plan

Learning Activities:

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3. Critical Thinking Activities

Students are involved in more challenging discussions and activities at grade level that are related to higher-order thinking skills according to the revised Bloom's Taxonomy as below:

Applying

Students can apply their knowledge on mathematical concepts to other contexts in their lives after reading. Student application of their knowledge will be demonstrated during the class activities, such as discussion, presentation, peer-review, and problem-solving.

Analyzing

Students can compare and contrast different methods for solving problems. Students also analyze different types of problems without a clearcut solution laid out for them. Finally, students will also analyze their peers board work and presentations. Students will gain an appreciation for peer-review, which is a fundamental element of both mathematics and science.

Evaluating

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Creating

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The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 4: Complex Numbers, Euler's Number, and Logarithms, Unit 1 Complex Numbers

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Define i
- State basic properties of i
- Define complex numbers
- Perform basic operations on complex numbers

Understanding(s):
Students will understand …

- How the solution to the equation $x^2+1=0$ may be represented by $i=\sqrt{-1}$.
- The basic properties of i.
- The definition of complex numbers $\{b+ai | i=\sqrt{-1} \text{ and } a,b \in \Re\}$
- Basic operations on complex numbers

Essential Question(s):

- How do we solve the equation $x^2+1=0$?

Knowledge:
Students will know …

- How to work with i.
- How to discuss complex numbers.
- How to perform basic operations on complex number.

Skills:
Student will be able to:

- Define i
- State basic properties of i
- Define complex numbers
- Perform basic operations on complex numbers

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30%

Other Evidence:

The following will also be observed, recorded, and considered for the final grade of students in each lesson activity
Motivation
Engagement

of student grades. • Comprehension (true/false, definitions, identifying topics and themes, etc.) • Solving pure mathematical problems as well as word problems. • Discussions and presentations • Group project that involves research and report writing • Homework assignments	Collaboration Communication pattern among peers and with the teacher Reactions Respect to others and different opinions

Stage 3 – Learning Plan

Learning Activities:

In this course, students are involved in a variety of class activities to understand mathematics at a deeper level, to transfer their knowledge to other contexts, and to improve their skills of working with mathematics in the form of discussion, presentation, and interaction. In so doing, students demonstrate their ability to use English mathematical language and notation appropriate to their grade level. The following is a summary of lesson activities for the course.

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3. Critical Thinking Activities

Students are involved in more challenging discussions and activities at grade level that are related to higher-order thinking skills according to the revised Bloom's Taxonomy as below:

Applying

Students can apply their knowledge on mathematical concepts to other contexts in their lives after reading. Student application of their knowledge will be demonstrated during the class activities, such as discussion, presentation, peer-review, and problem-solving.

Analyzing

Students can compare and contrast different methods for solving problems. Students also analyze different types

of problems without a clearcut solution laid out for them. Finally, students will also analyze their peers board work and presentations. Students will gain an appreciation for peer-review, which is a fundamental element of both mathematics and science.

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The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 4, Unit 2 Euler's Number and Natural Logarithms

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Define e^x and use the definition to estimate e
- Understand the rules of Natural Logarithms

Understanding(s):
Students will understand …

- How to calculate basic logarithms of any base.
- How to calculate common and natural logarithms.
- How to estimate e.
- The importance of e^x and natural logarithms.

Essential Question(s):

- How do we calculate compounding interest?
- What is the relationship between compounding interest and physics?

Knowledge:
Students will know …

- How to work with e^x.
- How to discuss logarithms.

Skills:
Student will be able to:

- Define e^x and use the definition to estimate e
- Understand the rules of Natural Logarithms

Stage 2 - Assessment Evidence

Performance Task(s):

To assess student progress made in this course, student work in the following activities will be clearly recorded and evaluated according to criteria, rubrics, and the teacher's discretion. Homework assignments will be given 10% and all the others will be given 30% of student grades.

- Comprehension (true/false, definitions, identifying topics and themes, etc.)
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Other Evidence:

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Engagement
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Reactions
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Learning Activities:

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The Asian International School
Unit Backward Design
Mathematics, TOEFL Beginner, 2018-2019
Chapter 4, Unit 3 Functions with and

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Understand the derivative involving e and ln
- Understand Euler's Formula
- Understand Euler's Identity
- Understand the importance of the above topics to our modern world

Understanding(s):
Students will understand …

- Derivatives involving e and ln
- Euler's Formula
- Euler's Identity

Essential Question(s):

- What is the most beautiful equation in all of mathematics?
- How do we begin to understand subjects like Quantum Mechanics?

Knowledge:
Students will know …

- $e^{ix} = \cos x + i \sin x$
- $e^{i\pi} + 1 = 0$
- How to find derivatives involving e and ln

Skills:
Student will be able to:

- Understand the derivative involving e and ln
- Understand Euler's Formula
- Understand Euler's Identity
- Understand the importance of the above topics to our modern world

Stage 2 - Assessment Evidence

Performance Task(s):

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